Dynamic Response of Laminated Composite Plates

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Theme

THE classical method of separation of variables combined with the Mindlin-Goodman procedure[1] is employed to analyze the dynamic response of an infinitely long simply-supported composite plate under a uniform dynamic pressure, p_0, on the upper surface of the plate. Two cases of dynamic pressures are considered; 1) the magnitude of the uniform pressure remains unchanged, and 2) the magnitude of the uniform pressure increases linearly as a function of time and then remains constant. Numerical results for eight layer granite/epoxy and glass/expoxy composites with $(0/0/\theta/-\theta)_s$ and $(\theta/-\theta/0/0)_s$ stacking sequences are evaluated. Dynamic response characterized in terms of normal displacement, bending stress, interlaminar shear stress, and interlaminar normal stress are evaluated and compared with the corresponding quantities obtained statically (i.e., a dynamic load factor is established). It is observed that the dynamic values are consistently twice the corresponding static values for Case 1, and varies from one to two times the corresponding static values for Case 2.

Contents

The equations of motion, including transverse shear deformation and rotary inertia, for an infinitely long laminated composite plate under cylindrical bending are given in Ref. 2. The solution to the governing equations in terms of separation of variables in conjunction with the Mindlin-Goodman procedure[1] is also presented in Ref. 2. In the present paper the interlaminar shear stress τ_{xz} is determined by integrating the equation of motion

$$\partial\sigma_{xx}{}^{(k)}/\partial x+\partial\tau_{xz}{}^{(k)}/\partial z=\rho z(\partial^2\psi_x/{}_{\partial t}2) \tag{1}$$

with respect to z using the condition $\tau_{xz}(\pm h/2,t)=0$ and the condition of continuity at the interfaces. Once $\tau_{xz}{}^{(k)}$ is determined the interlaminar normal stress $\sigma_{zz}{}^{(k)}$ can also be determined by integrating the equations of motion

$$\partial\sigma_{zz}{}^{(k)}/\partial z+\partial\tau_{xz}{}^{(k)}/\partial x=\rho\,(\partial^2 w/\partial t^2 \tag{2}$$

In Eqs. (1) and (2) x, z, σ_{xx}, σ_{zz}, ρ, ψ_x, and w denote the in-plane coordinate perpendicular to the supported sides, thickness coordinate, inplane normal stress, interlaminar normal stress, density per unit volume, x rotation of the laminate mid-plane, and transverse deflection of the laminate, respectively. The superscript k denotes the kth layer of the laminate.

Dynamic load factors (DLF) are shown in Table I for graphite-epoxy angle-ply laminates having the following

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Index categories: Structural Composite Materials (including Coatings); Structural Dynamic Analysis.

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Table 1 Dynamic load factor

	Graphite-epoxy $(0/0/\theta/-\theta)_s$						
θ	0	15	30	45	60	75	90
$w^{D/}{}_w ST$	2.002	2.002	2.003	2.000	2.004	2.005	2.002
$\sigma_{xx}{}^{D/}\sigma_{xx}ST$	2.048	2.048	2.054	2.070	2.050	2.051	2.051
	Graphite-epoxy $(\theta/-\theta/0/0)_s$						
θ	0	15	30	45	60	75	90
$w^{D/}{}_w ST$	2.002	2.004	2.009	2.010	2.006	2.005	2.006
$\sigma_{xx}{}^{D/}\sigma_{xx}ST$	2.048	2.050	2.064	2.077	1.911	2.048	2.060

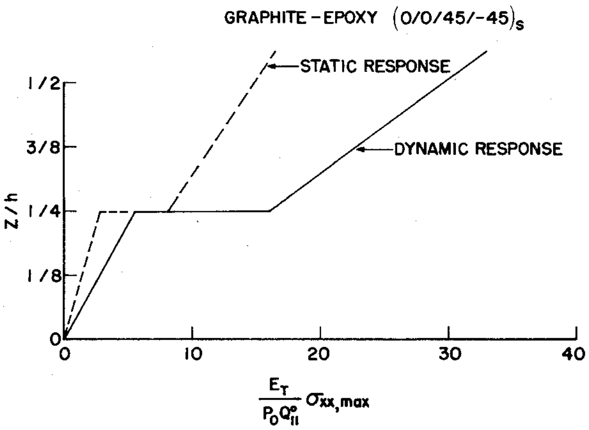

Fig. 1 Maximum bending stress distribution through laminate thickness for $(0/0/45/\text{-}45)_s$ laminate.

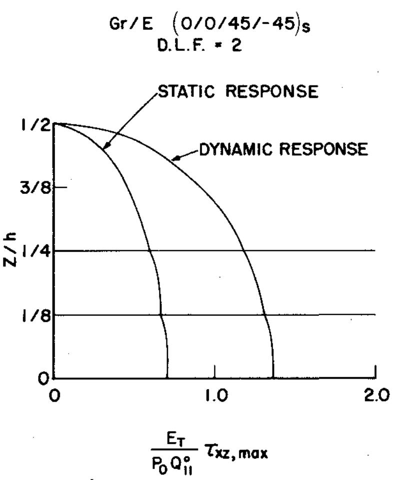

Fig. 2 Maximum interlaminar shear stress distribution through laminate thickness for $(0/0/45/\text{-}45)_s$ graphite-epoxy laminate.

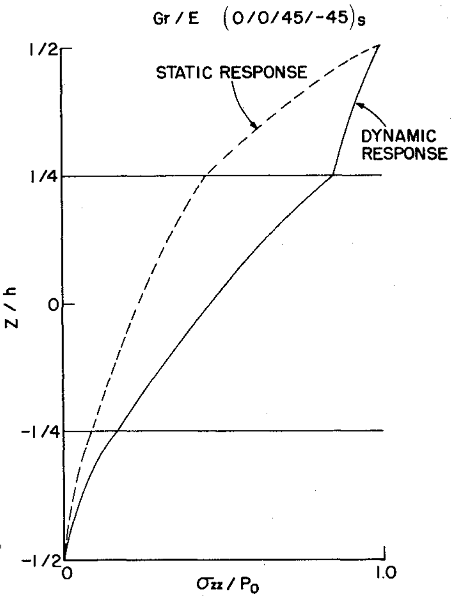

Fig. 3 Maximum interlaminar normal stress distribution through laminate thickness for (0/0/45/-45)$_s$ graphite-epoxy laminate.

unidirectional properties:

$E_L = 20 \times 10^6$psi, $E_T = 10^6$psi
$\nu_{LT} = \nu_{TT} = 0.25$,
$G_{LT} = 0.6 \times 10^6$ psi, $G_{TT} = 0.5 \times 10^6$psi

where L and T are the directions parallel and normal to the fibers, respectively, and ν_{LT} is the Poisson ratio measuring transverse strain under uniaxial normal stress parallel to the fibers. The value of k, the transverse shear correction factor, is taken as the classical value $\pi/(12)$. The angle θ in each ply is the angle between the x-direction and the direction of fiber orientation. It is also assumed that $h/L = 0.05$ where h and L represent the thickness and the span length of the plate, respectively. The maximum dynamic and static values of σ_{xx}, τ_{xy}, and σ_{zz} as a function of the thickness coordinate z are shown in Fig. 1-3, respectively, for a (0/0/45/-45)$_s$ graphite-epoxy laminate, where $Q_{11}°$ is the stiffness in the x direction of the 0 degree ply. In Fig. 4 the DLF is plotted for the same laminate as a function of the loading time T_1 where

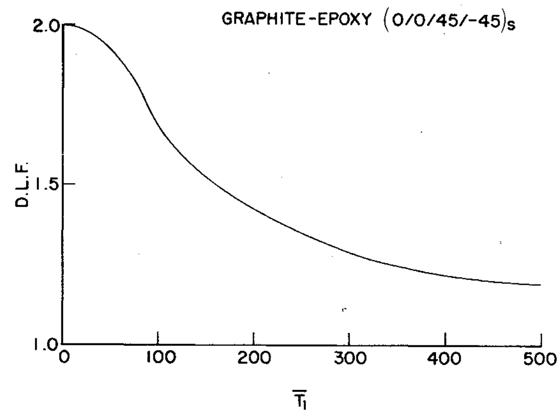

Fig. 4 Dynamic load factor as a function of $\bar{T}_1$ for (0/0/45/-45)$_s$ graphite-epoxy laminate.

$$\bar{T}_1 = (E_T/\rho)^{1/2}(t_1/h)$$

and t_1 is the actual loading time.

References

[1]Mindlin, R.D. and Goodman, L.E., "Beam Vibrations with Time-Dependent Boundary Conditions," *Journal of Applied Mechanics,* Vol. 17, Dec. 1950, pp. 377-380.

[2]Sun, C.T. and Whitney, J.M., "Forced Vibrations of Laminated Composite Plates in Cylindrical Bending," *Journal of Acoustical Society of America,* May 1974, pp. 1003-1008.